**NON-PARAMETRIC TEST**

**Wilcoxon Matched Pair Signed-Ranked Test**

Wilcoxon Signed-Ranked test is a non-parametric test. It is based on both the direction and magnitude of differences within each pairs of related samples.

**Case-I:** Small sample case when n 25.

**Problem:** To test,

**Null hypothesis (H0):** F (X) = F(Y) ⇒(y); there is no significance difference between median of X and median of Y.

**Alternative hypothesis H1:** F (X) F(Y) ⇒; Two-tailed test

**H1:** F (X) >F (Y) ⇒; One-tailed test

**H1:** F (X) <F (Y) ⇒; One-tailed test

**Test statistic:** Under H0, test statistic is

T = Minimum of (S + or S-)

Where,

= - = -

S + = Sum of ranks of positive d i

S – = Sum of rank of negative d i

**Critical region:** Next for a pre-assigned level of significance and corresponding size n. We from the table the critical value is for one tailed and for two tailed test.

**Decision:** If T> ; we accept H0. Otherwise reject H0.

**Case-II:** Small sample case when n 25.

In case of large sample the sampling distribution of **T** is approximately normal with mean = and variance =

**Test statistic:** Under H0, test statistics is given by

Z=

**Critical region:** Next for a pre-assigned level of significance, the probability (p0) is associated with the values as extreme as z, we obtained from z- table is

p0 = p ().

**Decision:** For one-tailed test, if p0 , we reject H0. Otherwise reject H0.

For two-tailed test, if 2 p0 , we reject H0. Otherwise reject H0.

**Numerical problem**

**Example (1):** In a study of nutritional qualities of fast food, two food technologists measured the amount of fat content in random sample of 7 hamburgers of a particular restaurant chain.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Food technologists | Hamburgers | | | | | | |
| X | 64 | 74 | 92 | 74 | 68 | 84 | 90 |
| Y | 72 | 81 | 60 | 70 | 72 | 74 | 82 |

Analyze the data by using Wilcoxon matched pairs signed rank test.

**Solution:** Here,

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | y | d = x - y | Rank of |d| | Rank of +ve d | Rank od -ve d |
| 64 | 72 | -8 | 4.5 | - | 4.5 |
| 74 | 81 | -7 | 3 | - | 3 |
| 92 | 60 | 32 | 7 | 7 | - |
| 74 | 70 | 4 | 1.5 | 1.5 | - |
| 68 | 72 | -4 | 1.5 | - | 1.5 |
| 84 | 74 | 10 | 6 | 6 | - |
| 90 | 82 | 8 | 4.5 | 4.5 | - |
|  |  |  |  | = 19 | = 9 |

n = No. of effective samples = 7

Now,

**Problem:** To test,

**Null hypothesis (H0):** F (X) = F(Y) ⇒(y); there is no significance difference between median of X and median of Y.

**Alternative hypothesis H1:** F (X) F(Y) ⇒; Two-tailed test

**Test statistic:** Under H0, test statistic is

T = Minimum of (S + or S-) = 9

**Critical region:** Next for a pre-assigned level of significance = 0.05 and corresponding effective sample size n = 7. We from the Wilcoxon-Matched table the critical value is

= = = 2

**Decision:** Since T> ; we accept H0.

**Conclusion:** There in no significance difference between median of sample X and median of sample Y.

**Example (2):** The following give the corrosion effects in various soils for coated and uncoated steel pipes.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Uncoated Pipes | 48 | 43 | 38 | 62 | 75 | 56 | 58 | 45 | 56 | 38 | 71 | 52 | 59 | 63 | 40 |
| 58 | 45 | 31 | 58 | 45 | 60 | 42 | 38 | 51 | 59 | 35 | 40 | 58 | 70 | 43 |
| Coated Pipes | 55 | 48 | 29 | 54 | 73 | 58 | 60 | 41 | 50 | 36 | 65 | 56 | 54 | 60 | 42 |
| 60 | 41 | 30 | 54 | 47 | 61 | 40 | 38 | 53 | 56 | 35 | 43 | 60 | 66 | 42 |

Test the null hypothesis that the corrosion effects for coated and uncoated steel pipes are the same.

**Solution:** Here,

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | y | d = x - y | Rank of |d| | Rank of +ve d | Rank od -ve d |
| 48 | 55 | -7 | 26 |  | 26 |
| 43 | 48 | -5 | 22.5 |  | 22.5 |
| 38 | 29 | 9 | 28 | 28 |  |
| 62 | 54 | 8 | 27 | 27 |  |
| 75 | 73 | 2 | 8.5 | 8.5 |  |
| 56 | 58 | -2 | 8.5 |  | 8.5 |
| 58 | 60 | -2 | 8.5 |  | 8.5 |
| 45 | 41 | 4 | 19 | 19 |  |
| 56 | 50 | 6 | 24.5 | 24.5 |  |
| 38 | 36 | 2 | 8.5 | 8.5 |  |
| 71 | 65 | 6 | 24.5 | 24.5 |  |
| 52 | 56 | -4 | 19 |  | 19 |
| 59 | 54 | 5 | 22.5 | 22.5 |  |
| 63 | 60 | 3 | 15 | 15 |  |
| 40 | 42 | -2 | 8.5 |  | 8.5 |
| 58 | 60 | -2 | 8.5 |  | 8.5 |
| 45 | 41 | 4 | 19 | 19 |  |
| 31 | 30 | 1 | 2 | 2 |  |
| 58 | 54 | 4 | 19 | 19 |  |
| 45 | 47 | -2 | 8.5 |  | 8.5 |
| 60 | 61 | -1 | 2 |  | 2 |
| 42 | 40 | 2 | 8.5 | 8.5 |  |
| 38 | 38 | 0 | - |  |  |
| 51 | 53 | -2 | 8.5 |  | 8.5 |
| 59 | 56 | 3 | 15 | 15 |  |
| 35 | 35 | 0 | - |  |  |
| 40 | 43 | -3 | 15 |  | 15 |
| 58 | 60 | -2 | 8.5 |  | 8.5 |
| 70 | 66 | 4 | 19 | 19 |  |
| 43 | 42 | 1 | 2 | 2 |  |
|  |  |  |  | = 262 | = 144 |

n = No. of effective sample size = 28

= = = 203

= = = 1928.5

= = 43.9147

T = Minimum of (S + or S-) = 144

Now,

**Problem:** To test,

**Null hypothesis (H0):** F (X) = F(Y) ⇒(y); there is no significance difference between the corrosion effects for coated and uncoated steel pipes.

**Alternative hypothesis H1:** F (X) F(Y) ⇒; Two-tailed test

**Test statistic:** Under H0, test statistics is given by

Z= = = - 1.34

**Critical region:** Next for a pre-assigned level of significance = 0.05, the probability (p0) is associated with the values as extreme as z, we obtained from z- table is

p0 = p () = p () = p () = 0.0901

2 p0 = 2 0.0901 = 0.1802

**Decision:** Since 2 p0 >, we accept H0.

**Conclusion:** There is no significance difference between the corrosion effects for coated and uncoated steel pipes.